

# **Measure the Universe: All Units, Constants** (v4.1 added volt-amp)

No formulas. Just one unit per phenomenon to be measured. Named fundamental relationships for conceptual understanding of the units. Plus constants, because they are constant like units.

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# 1: Basic Units

## Energy, scalar, conserved in 1<sup>st</sup> Law of Thermodynamics

$$E \text{ in Joules} = J = N \cdot m = \text{kg} \cdot \frac{\text{m}^2}{\text{s}^2} = \text{Watt} \cdot \text{s} = \text{Volts} \cdot \text{Amps} \cdot \text{seconds}$$

$$\text{Energy} = \text{force} \cdot \text{distance} = \text{mass} \cdot \text{velocity}^2 = \text{power} \cdot \text{time} = \text{potential} \cdot \text{Current} \cdot \text{time}$$

$$eV = 1.602 \times 10^{-19} J \quad || \quad BTU = 1055 J \quad | \quad | \text{Calorie} = \text{kilocalorie} = \text{foodcalorie} = 4184 J$$

Joules are also used to measure total internal energy and enthalpy.

## Power, scalar, also conserved because Energy must be conserved

$$P \text{ in Watts} = W = \frac{J}{s} = \frac{N \cdot m}{s} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} = \text{Volts} \cdot \text{Amps} = I^2 \cdot R = \frac{U^2}{R} \quad || \quad HP = 745.7 W$$

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \text{potential} \cdot \text{current} = \text{current}^2 \cdot \text{resistance} = \frac{\text{potential}^2}{\text{resistance}}$$

## Efficiency, Ratio of Useful Energy Obtained to Energy Consumed

$$\eta = [ \text{dimensionless, usually } \% ] = \frac{E_{out}}{E_{in}} = \frac{P_{out}}{P_{in}} = \frac{E_{in} - E_{loss}}{E_{in}} = \frac{P_{out} - P_{loss}}{P_{in}}$$

Efficiency is *the only place* where energy or power – in any form – can be “lost.” Even if “lost” in efficiency, energy and power become heat or some other form of energy or power (yeah OK, or  $E=mc^2$ ).

## Mass, m, scalar

$$m \text{ in kilograms} = \text{kg} \quad || \quad \text{pound}_{on-earth} = \text{lb} = 0.4536 \text{kg} \quad | \quad Mg = \text{"tonne"}$$

## Length or Distance, scalar

$$\text{meter} = m \quad || \quad \text{inch} = 0.0254m \quad | \quad \text{foot} = 0.3048m \quad (\text{both inch and foot are exact, by definition})$$

## Area, scalar, squared length

$$m^2 = \text{square meter} = \text{length}^2 \quad || \quad \text{hectare} = 10^4 m^2 \quad || \quad \text{acre} = 4046.86 m^2$$

## Volume, V, scalar, cubed length

$$m^3 = \text{cubic meter} = 1000 \text{ liters } (\sim \text{thousand kg water})$$

$$\text{gallon} = 0.0037854 m^3 = 128 \text{ fluid ounces}$$

## Time

seconds always for time

## Force, vector, conserved in Newton's 2<sup>nd</sup> Law

$$\vec{F} \text{ in Newtons} = N = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \quad || \quad \text{pound}_{force} = \text{lbf} = 4.448 N$$

## Momentum, vector, conserved in Newton's 2<sup>nd</sup> Law

$$\vec{p} = m \cdot \vec{v} = \text{kg} \cdot \frac{\text{m}}{\text{s}}$$

## Frequency, Cycles, or Revolutions per Time or Rotational Speed

$$f \text{ in Hertz} = \frac{1}{s} \text{ for cycles and } \frac{\text{rev}}{s} \text{ for rotational speed} \quad || \quad \omega = 2\pi \cdot \text{freq} = 2\pi \cdot \frac{\text{rev}}{s} \text{ radians}$$

## Angle

$$\text{radian} \Rightarrow \frac{2 \cdot \pi \text{ radians}}{\text{full circle}} \quad || \quad \text{degree} = \frac{\pi}{180} \text{ radians}$$

“Solid angle” = fractional portion of surface of sphere with radius of one, [see Nuclear 5 slide 75...](#)

## Torque

$$\tau \text{ in } N \cdot m = \text{Newton} \cdot \text{meter} \Rightarrow [\text{Joule}]$$

the Energy (or Joule) relationship makes sense when considering the power of a rotating shaft:

$$P_{\text{shaft}} = \tau \times \omega [\text{Watts}]$$

## Pressure, p or P, scalar (vector is considered normal to the area on which it acts)

$$p \text{ in Pascals} = Pa = \frac{N}{m^2} = \frac{kg}{m \cdot s^2} \Rightarrow \text{also } \frac{J}{m^3} \quad || \quad \text{atmosphere} = 101325 Pa \quad || \quad PSI = 6895 Pa$$

$$mmH_2O = 9.81 Pa \quad || \quad mmHg = 133.3 Pa$$

## Temperature, T, scalar

T in Kelvins. Water, at  $P_{\text{water}} = 1.013 \times 10^5 Pa$ , melts at 273.15 Kelvins and boils at 373.15 Kelvins.

# E1: Electromagnetism Units

**Electric Potential, scalar, energy per charge**

$$U \text{ in Volts} = V = \frac{E_{\text{potential}}}{q} = \frac{J}{C} = \frac{kg \cdot m^2}{s^3 \cdot A} = A \cdot \Omega = \frac{W}{A} = \vec{E} \cdot m$$

**Electric Field, vector, potential per distance, and Force, gradient of potential**

$$\vec{E} = \frac{\text{Volts}}{\text{meter}} = \frac{kg \cdot m}{s^3 \cdot A} = \frac{\text{Newtons}}{\text{Coulomb}} = \frac{N}{C}$$

**Electric Flux, vector with scalar, “electric field through a surface”**

$$\vec{\Phi}_{\text{electric}} = \vec{E} \cdot \vec{S} = E \cdot S \cdot \cos \theta \Rightarrow \text{in } V \cdot m = \frac{V}{m} \cdot m^2 = \frac{N \cdot m^2}{C} = \frac{kg \cdot m^3}{s^3 \cdot A}$$

**Electric Charge, scalar, energy per potential, quantized**

$$q \text{ in Coulombs} = C = \frac{\text{Joules}}{\text{Volt}} = A \cdot s$$

$e = 1.602 \times 10^{-19} C = \text{elementary charge (proton and electron charge)}$

**Electric Current, charge per time**

$$I \text{ in Amps} = A = \frac{\text{Coulombs}}{s} = \frac{C}{s}$$

**Current Density, vector, charge per time per area**

$$\vec{j} \text{ in } \frac{\text{Amps}}{m^2} \Rightarrow \text{direction is of positive charged particles}$$

**Magnetic Field Strength, vector (AKA “Magenetic Field”)**

$$\vec{H} = \frac{\text{Amps}}{\text{meter}} = \frac{A}{m} = \frac{\vec{B}}{\mu}$$

**Magnetic Flux, vector with scalar**

$$\vec{\Phi}_B = \vec{B} \cdot \vec{S} = B \cdot S \cdot \cos \theta \Rightarrow \text{direction is normal to the surface}$$

$$\text{Weber} = \text{Wb} = \Omega \cdot C = V \cdot s = H \cdot A = T \cdot m^2 = \frac{J}{A} = \frac{kg \cdot m^2}{s^2 \cdot A}$$

**Magnetic Flux Density, vector (also AKA “Magnetic Field”)**

$$\vec{B} = \text{Tesla} = \frac{N}{m \cdot A} = \frac{kg}{s^2 \cdot A} = \mu \cdot \vec{H} \Rightarrow \text{is much higher inside magnetic material}$$

$$\text{Tesla} = T = \frac{\text{Wb}}{m^2} = \frac{V \cdot s}{m^2} = \frac{N}{A \cdot m} = \frac{H \cdot A}{m^2} = \frac{kg}{C \cdot s} = \frac{N \cdot s}{C \cdot m} = \frac{kg}{A \cdot s^2}$$

## E2: Electric Circuit Units

**Resistance, R, scalar**

$$Ohm = \Omega = R = \frac{I}{U} = \frac{Amps}{Volt} = \frac{A}{V}$$

**Impedance, Z, complex phasor, where X is Reactance**

$$Z = R + j \cdot X \text{ where } X = X_L - X_C = X_{inductive} - X_{capacitive}$$

**Inductance, scalar: L is inductance, X<sub>L</sub> is inductive reactance**

$$Henry = H = \frac{V \cdot s}{A} = \frac{kg \cdot m^2}{s^2 \cdot A^2} = \frac{N \cdot m}{A^2} = \frac{kg \cdot m^2}{Coulomb^2} = \frac{J}{A^2} = \frac{T \cdot m^2}{A} = \frac{Weber}{A} = \frac{\Omega}{Hz} = \Omega \cdot s = \frac{s^2}{F}$$

$$v(t) = L \cdot \frac{di}{dt}$$

$$Z_{inductor} = Z_L = j \cdot X_L = j \cdot \omega \cdot L = \omega \cdot L \cdot e^{j \cdot \frac{\pi}{2}} = j \cdot 2\pi \cdot freq \cdot L \text{ and } L = \frac{X_L}{2\pi \cdot freq}$$

**Capacitance, scalar: C is capacitance, X<sub>C</sub> is capacitive reactance**

$$Farad = F = \frac{Coulombs}{Volt} = \frac{A \cdot s}{V} = \frac{J}{V^2} = \frac{N \cdot m}{V^2} = \frac{C^2}{J} = \frac{s}{\Omega} = \frac{1}{\Omega \cdot Hz} = \frac{s^2}{H} = \frac{s^4 \cdot A^2}{m^2 \cdot kg} = \frac{s^2 \cdot C^2}{m^2 \cdot kg}$$

$$i(t) = C \cdot \frac{dv(t)}{dt}$$

$$Z_{capacitor} = Z_C = -j \cdot X_C = -j \frac{1}{\omega \cdot C} = \frac{1}{\omega \cdot C} \cdot e^{j \cdot (-\frac{\pi}{2})} = -j \cdot \frac{1}{2\pi \cdot freq \cdot C} \text{ and } C = \frac{1}{2\pi \cdot freq \cdot X_C}$$

**Magnetic Reluctance, opposition to magnetic flux, inverse is Magnetic Permeance**

$$\mathcal{R} = \frac{\mathcal{F}}{\Phi_B} = \frac{MMF}{Magnetic Flux} = \frac{A \cdot turns}{Weber} = \frac{turns}{Henry} \quad || \quad \mathcal{P} = \frac{1}{\mathcal{R}}$$

Possibly add an image of **IPSE book Table C.1** here. Very useful for magnetic circuits.

**Apparent Power, scalar: S is apparent power, P is real power, Q is reactive power**

$$volt \cdot amp = VA = V_{RMS} \times I_{RMS} = S = \sqrt{(real\ power)^2 + (reactive\ power)^2} = \sqrt{P^2 + Q^2}$$

$$P = \sqrt{S^2 - Q^2} = S \times power\ factor = S \times \cos \phi = S_{DC}$$

## E3: Electrical Properties of Materials and Fluids, and Constants

### Resistivity, Conductivity of a material

$$\rho = \Omega \cdot m = R \frac{A}{l} \Rightarrow R = \rho \cdot \frac{l}{A} \Rightarrow \text{resistivity}$$

$$\sigma = \rho^{-1} = \frac{1}{\Omega \cdot m} = \frac{l}{R \cdot A} \Rightarrow R = \frac{l}{\sigma \cdot A} \Rightarrow \text{conductivity}$$

### Permittivity, Vacuum AKA “distributed capacitance of the vacuum”

$$\epsilon_0 = 8.854 \times 10^{-12} \frac{\text{Farads}}{\text{meter}}$$

### Relative Permittivity and Permittivity of a Medium

$$\epsilon_{\text{relative}}(\omega) = \frac{\epsilon(\omega)}{\epsilon_0} \quad || \quad \epsilon(\omega) = \epsilon_{\text{relative}}(\omega) \cdot \epsilon_0 \quad || \quad \epsilon_{\text{relative}} \text{ of teflon is } 2.1 \text{ [dimensionless]} \quad || \text{ also } \kappa$$

### Magnetic Permeability, Vacuum (“Magnetic Permeability” = “Permeability”)

$$\mu_0 = 4 \cdot \pi \times 10^{-7} \frac{\text{Henry}}{\text{meter}} = 1.257 \times 10^{-6} \frac{H}{m} \Rightarrow \frac{H}{m} = \frac{N}{A^2}$$

### Relative Permeability and Permeability of a Medium

$$\mu_{\text{relative}} = \frac{\mu}{\mu_0} \quad || \quad \mu = \mu_{\text{relative}} \cdot \mu_0 \quad || \quad \mu_{\text{relative}} \text{ of iron is } \sim 200,000 \text{ [dimensionless]}$$

### Planck Constant, relates photon frequency to photon energy

$$h = 6.626 \times 10^{-34} J \cdot s \quad || \quad E_{\text{photon}} = h \cdot f_{\text{photon}}$$

### Speed of Light

$$c_0 = 3.0 \times 10^8 \frac{m}{s} = \frac{1}{\sqrt{\mu_0 \cdot \epsilon_0}}$$

## E4: Electromagnetism, Named Laws and Equations

Maxwell's equations go here.

### Lorentz Force Law

$$\vec{F}_{\text{electromagnetic}} = \vec{F}_{\text{electric}} + \vec{F}_{\text{magnetic}} = q \cdot (\vec{E} + \vec{v} \times \vec{B})$$

### Coulomb's Law

$$F_{\text{Coulomb Law}} = \frac{1}{4 \cdot \pi \cdot \epsilon_0} \frac{q_1 \cdot q_2}{r^2}$$

## F1: Fluids, Properties of Fluids and Materials

Density, mass per volume:

$$\rho = \frac{kg}{m^3}$$

Density inverse is specific volume:

$$v = \rho^{-1} = \frac{m^3}{kg}$$

Speed of Sound,  $c$ , in Ideal Gases and Air

$$c = \sqrt{\frac{K_S}{\rho}} = \sqrt{\lambda \cdot \frac{P}{\rho}} \Rightarrow c_{air} = 20.05 \cdot \sqrt{T} \frac{m}{s}$$

Viscosity (dynamic viscosity and kinematic viscosity)

$$\mu = Pa \cdot s = \frac{N \cdot s}{m^2} = \frac{kg}{m \cdot s} \Rightarrow \text{dynamic viscosity } \mu = \rho \cdot \nu$$

$$\nu = \frac{m^2}{s} \Rightarrow \text{kinematic viscosity } \nu = \frac{\mu}{\rho} \quad || \quad \text{stokes} = \frac{cm^2}{s} = 0.0001 \frac{m^2}{s}$$

Bulk Modulus,  $K$ : Resistance to Compression of a substance.

$$K = -V \cdot \frac{dP}{dV} = \rho \frac{dP}{d\rho}$$

$$K_S = \lambda \cdot P \text{ (isentropic)} \quad || \quad K_T = P \text{ (isothermal)}$$

## F2: Fluids, Named Laws and Equations

Need Navier-Stokes etc. here



# T1: Thermodynamics, Materials, Fluids, Constants

## Heat Energy

$Q$  in Joules

## Heat Energy Flow

$$\dot{Q} \text{ in } \frac{\text{Joules}}{\text{s}} = \text{Watts}$$

**Enthalpy, internal energy plus “work required to establish the system’s physical dimensions”**

$$H \text{ in Joules} = U + p \cdot V$$

**Internal Energy, energy contained within the system excluding kinetic and potential energy**

$U$  in Joules

## Heat Flux, heat flow per area

$$q = \frac{\dot{Q}}{\text{Area}} \text{ in } \frac{\text{Watts}}{\text{m}^2} \Rightarrow \text{in this reference, always } \dot{Q} \text{ in Watts or } Q \text{ in Joules. No "little } q."$$

**Specific heat (often given for either constant pressure or constant volume)**

$$c_P \text{ and } c_V \text{ are both measured in } \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$c_P = c_V + R_{\text{specific}}$$

$$\lambda = \frac{c_P}{c_V} = \text{heat capacity ratio}$$

$c_P$  is always larger [almost always larger, water?] than  $c_V$  conceptually because “constant volume” means the pressure increases as energy is added and “helps” heat the substance. “Constant *pressure*” implies the gas is allowed to expand while energy is added and expanding materials cool. Thus constant pressure requires more energy per temperature to make up for the cooling that *would have* happened.

## Thermal Conductivity, heat transfer of a material

$$\kappa = \frac{\dot{Q}}{\text{Area}} \cdot \frac{\text{thickness}}{\Delta T} \text{ in } \frac{\text{Watts}}{\text{m} \cdot \text{K}} = \frac{\text{Watts}}{\text{m}^2} \cdot \frac{\text{m}}{\Delta T} \Rightarrow \text{sometimes } \lambda \text{ or } k \Rightarrow \dot{Q} = \kappa \cdot \frac{\text{Area} \cdot \Delta T}{\text{thickness}}$$

## Heat Transfer Coefficient, of a surface

$$h = \frac{\dot{Q}}{A} \cdot \frac{1}{\Delta T} = \frac{\text{Watts}}{\text{m}^2 \cdot \text{K}} \Rightarrow \dot{Q} = h \cdot A \cdot \Delta T \text{ and } h_{\text{surface}} = \kappa_{\text{surface material}} \cdot \text{thickness}$$

## Steffan-Boltzmann Constant, radiant heat of a black body

$$\sigma = 5.670 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$$

can also be directly related to the Boltzmann constant, Planck constant, and  $c_0$  (= speed of light)

**Boltzmann Constant, relates average kinetic energy of particles in a gas to thermodynamic temperature**

$$k_B = 1.381 \times 10^{-23} \frac{J}{K}$$

**Gas Constant and Specific Gas Constant**

$$R = R_{molar} = 8.314 \frac{J}{K \cdot mol}$$

$$R_{specific} = \frac{R}{M} \text{ unit is } \frac{J}{kg \cdot K} \left( \text{note: } M \text{ usually given in } \frac{g}{mol} \text{ so must convert to } \frac{kg}{mol} \right)$$

**Ideal Gas**

$$PV = nRT \text{ and } PV = m \cdot R_{specific} \cdot T$$

## **T2: Thermodynamics, Named Laws and Equations**

1. **Conservation of Energy.** The energy gained (or lost) by a system is equal to the energy lost (or gained) by its surroundings. Cannot produce work without energy input.
2. In a natural thermodynamic process, the sum of the entropies of the interacting thermodynamic systems never decreases. Heat does not spontaneously pass from a colder body to a warmer body. Cannot spontaneously convert thermal energy into mechanical work.
3. A system's entropy approaches a constant value as the temperature approaches absolute zero. With the exception of non-crystalline solids (glasses) the entropy of a system at absolute zero is typically close to zero.

named thermodynamics equations here

# N1: Nuclear Units

## Nuclear

## Energy

$$MeV = 1.602 \times 10^{-13}$$

## Area, Tiny

$$barn = 10^{-28}m^2$$

## Radioactivity, various forms

$$becquerel = Bq = \frac{1 \text{ decay}}{\text{second}} = \frac{1}{s} \Rightarrow \text{one 70kg human emits 8000 Bq normally from } \underline{\hspace{2cm}}$$

$$Curie Ci = 3.7 \times 10^{10} Bq = 37 GBq \quad || \quad Rutherford Rd = 10^6 Bq = 1 MBq$$

$$Gray = Gy = \frac{\text{Joule}}{kg} \text{ absorbed radiation energy} = \frac{m^2}{s^2}$$

$$Sievert = Sv = \frac{\text{Joule}}{kg} \Rightarrow \text{is Gray adjusted for stochastic health risk, LNT is "linear no threshold"}$$

$$\text{Normal in Belgium is } 0.005 \frac{Sv}{\text{year}} \text{ from } \underline{\hspace{2cm}}$$

Geez, how many v's are there? Volume, velocity, volts, Greek nu for viscosity. Some form of v for specific volume.